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Landscape shape adjusted compactness index for urban areas

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Abstract Scattered urban development leads to non-compact urban form. In this paper, I demonstrate that Index of Moment of Inertia is a useful metric to measure compactness. However, elongated political boundaries and natural restrictions severely distort the metric, rendering it less useful for monitoring urban development. I propose a landscape shape adjustment of this metric that retains some of the useful properties of the Index.

Keywords Compactness · Urban form · Landscape metrics

Introduction

It is of significant interest to characterize urban form and its change for various cities (e.g. Schneider and Woodcock 2008; Seto et al. 2011) and to study their effect, among other things, on air quality, transportation behaviour and relationships with environment (e.g. McCarty and Kaza 2015; Stone et al. 2007; Jones and Kammen 2014). One of the consistent findings in the literature is that sprawling urban form is costly and less sustainable (Carruthers and Ulfarsson 2003; Fan and Song 2009; Frumkin 2002). Urban sprawl is a multi-dimensional phenomena (Galster et al. 2001; Frenkel and Ashkenazi 2008; Schwarz 2010). There are number of ways to characterise urban sprawl including low population density, predominance of single family housing and discontinuous urban development. To monitor sprawling patterns, various indicators that rely on continuity, density, clustering and proximity have been proposed (e.g. Tsai 2005; Jaeger and Schwick 2014). One dimension of urban form that contributes to sprawl is dispersed fragmentary pattern. In this short note, I illustrate some challenges associated with measuring this pattern using a compactness measure derived from raster datasets and landscape metrics.

Conventionally compactness is defined for a single shape. Various metrics to characterise a compact shape have been proposed over the years (see Maceachren 1985; Gustafson 1998; Angel et al. 2010 for reviews). Many of them rely on comparing a property of the shape to that of the circle, the most compact 2D shape. For example, Schwartzberg (1966), Miller (1953), and Polsby and Popper (1991) rely upon the ratio of the perimeter of the shape to that of a corresponding circle of equal area. Reock (1961), uses the ratio of the areas of the shape and the minimum bounding circle. Zhao and Stough (2005) prescribe an overlap index that is a variant of an elongation index, that measures the ratio of a maximal overlap of the equal area circle to the shape. The

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maximal overlap is usually determined by an exhaustive search for a centre that is in the interior of the shape. Another commonly used shape compactness metric has been introduced by Boyce and Clark (1964), that uses equally spaced radials from the centroid and measuring the variation in the lengths of the radials to the perimeter. Angel et al. (2010) extend these concepts to characterize different notions of compactness of a shape by looking at other aspects such as dispersion, girth and range.

All these metrics are susceptible to issues of resolution and accuracy of measurement of perimeter and area as well proper treatment of holes and other non-simple features that are often found in urban areas. In particular, many of these metrics are designed for a single polygonal or contiguous/connected shapes. However, urban areas are often polynucleated and characterized by fragmentation especially at the edges (Batty 2001). Similarly, urban growth is often characterized by simultaneous processes of dispersion, coalescence and infill (e.g. Schneider and Woodcock 2008). Shape compactness metrics are local compactness metrics and for urban form. We need meso-level compactness metrics that take into account multiple distinct and discontinuous shapes at different distances. Unlike other landscape metrics to characterize urban form, central moments of the distribution of the shape compactness distribution (mean, variance etc.) are not sufficient to characterize the compactness of urban form and growth.

Landscape level compactness measures that are not averages or variances, also exist in the literature. For example, Zhou et al. (2016) use weighted area-perimeter ratio of the aggregated shape and the closeness between the parts of multi polygon as a measure of overall compactness. Huang et al. (2007) use the sum of the ratio of the perimeter of the patch to that of the Equal Area Circle (EAC) of the patch and normalise it by the square of the number of patches. Angel et al. (2010), defined Exchange Index (EI) as the ratio of the urban area that is covered by the EAC that is centred on the centre of gravity of the landscape. Li et al. (2013) proposed ratio of area moment of inertia (MI) of the shape to that of the EAC. Taubenböck et al. (2019) proposed a dispersion index (DI) that is based on number of patches (contiguous urban areas) and the largest patch index. While the former two are a measure of compactness, the latter is a measure of dispersion. In this paper, I focus on the Li et al. (2013) measure and compare it with the EI and DI. I demonstrate some challenges of this index by measuring the compactness of urban form in each county within the contiguous United States. I use this sample for its wide variety of urban form and political boundaries.

Index of moment of inertia as a measure of urban compactness

Two features of Li et al. (2013) measure makes it particularly attractive to measure urban dispersion. (1) MI of a collection is decomposable to its individual parts (2) Parallel Axis Theorem. If I_z is the MI of an area a, passing through an axis at centre, the MI at an axis d away from the center is $I_z + ad^2$. Furthermore, for a collection of areas K, the MI is $\sum_{k \in K}(I_z^k + a_k d_k^2)$, where d_k is the distance of each area a_k from the overall centroid and I_z^k is the MI of the individual part with respect to its own centroid. These properties make MI a useful metric for urban areas represented as rasters. The MI for each square (assuming that the raster resolution is the same in both x and y directions) with resolution s is $s^4/6$. Thus, the MI for the urban landscape S is

$$\sum_{i \in S}\left(\frac{s^4}{6} + d_i^2 s^2\right)\chi_S(i)$$

where $\chi_S(i) = 1$ when the cell i is urban, 0 otherwise. d_i is the distance of cell to the centroid of the urban area in the landscape.

The MI of the most compact shape, circle with the same area, is $A_u^2/(2\pi)$. Thus, the Index of Moment of Inertia (IMI) is

$$\mathrm{IMI}_u := \frac{A_u^2}{2\pi \sum_{i \in S} s^2(\frac{s^2}{6} + d_i^2)\chi_S(i)}$$

Since a raster can only approximate a circle, IMI is always $(0, 1)$ and is dimensionless.

This metric has been used to evaluate the spatial patterns of urbanisation (Kaza 2020), voting districts (Fan et. al 2015) and police districts (Bucarey et al. 2015). In a study of urban patterns in the US using this metric, Kaza (2020) found that urban areas in the South and the West are more fragmented and dispersed. In particular, counties in Florida and

California have both large number of urban patches and large average urban patch size. Large urban subdivision development in the green fields at the edges of cities, punctuated by undeveloped land or open spaces characterize these urban areas. Large central metro counties score well on this compactness metric compared to suburban and rural areas, which have both low and dispersed urban development patterns. Of all the large central metro counties, on average, counties in West North Central are most compact, while Mountain counties are least compact. In general, counties with higher degree of urbanization and centrality have higher compactness scores. There appears to be no discernible relationship between urban population density and IMI. Some dense places (e.g. New York, NY) have small IMI and some low density medium metropolitan areas have high compactness (e.g. Ronoke, VA) suggesting that the two indicators are orthogonal.

While IMI_u is a reasonable metric for measuring compactness, it has some problems associated with the landscape boundary and urban suitability. For example, the IMI for New York County (Manhattan) is 0.27, not because the urban form is spread out, but because the county boundaries are elongated (see Fig. 1). Another example is Multnomah County (Portland); it scores relatively modest on this metric belying its reputation for extensive growth management policies (Song and Knaap 2004). Furthermore, some county boundaries have water or undevelopable land on them, which constrains the urban from. Since we do not want to penalize these types of urban developments, we have to adjust the IMI to account for these exogenous factors.

Landscape shape adjustment

To correct for limitation of urban areas by geographical or political boundaries on the IMI, I first define a coverage index as A_u/A_L, where A_u is the urban area and A_L is the area of the developable landscape area. Both water area (including coastal waters and lakes) as well as land with high slopes are removed from the landscape boundary and the area A_L is calculated. To account for the non-compactness, I use the index of moment of inertia of this landscape shape for the adjustment, IMI_L. The adjusted urban compactness index is defined as

$$\mathrm{IMI}_{\mathrm{adj}} := \mathrm{IMI}_u^{\mathrm{IMI}_L*(1-\frac{A_u}{A_L})}$$

IMI_L as well as coverage index are less than 1. Since $\mathrm{IMI}_u < 1$, and because the adjustment factor is ≤ 1, $\mathrm{IMI}_{\mathrm{adj}} > \mathrm{IMI}_u$. Furthermore, $\mathrm{IMI}_{\mathrm{adj}}$ is always $(0, 1)$. Counties with higher coverage ratio and non-compact boundaries will receive larger adjustment, where as counties with low coverage ratio will receive a smaller adjustment. The adjustment factor is non-linear and would designate urban areas that are fully covering the landscape as the most compact with $\mathrm{IMI}_{\mathrm{adj}} = 1$, as the adjustment factor is 0, irrespective of the shape. Furthermore, the adjustment factor of the landscape shape is more relevant when the coverage index is

New York, NY $IMI_u = 0.27$ | Multnomah, OR $IMI_u = 0.53$ | Rock Island, IL $IMI_u = 0.20$

Fig. 1 Illustrations of external validity problems with IMI as a measure of compact urbanization

Table 1 Counties with the largest value of the compactness score and its change due to landscape adjustment

Census Division	IMI_u	IMI_{adj}	$IMI_{\Delta} := IMI_{adj} - IMI_u$
Pacific	Orange,California	Multnomah,Oregon	Jefferson,Washington
East North Central	Marion,Indiana	Marion,Indiana	Keweenaw,Michigan
West North Central	St. Louis,Missouri	St. Louis,Missouri	Pennington,South Dakota
Middle Atlantic	Kings,New York	Kings,New York	Suffolk,New York
New England	Kent,Rhode Island	Hampden,Massachusetts	Grand Isle,Vermont
East South Central	Shelby,Tennessee	Shelby,Tennessee	Perry,Kentucky
West South Central	Dallas,Texas	Orleans,Louisiana	Jefferson,Louisiana
Mountain	Salt Lake,Utah	Salt Lake,Utah	Shoshone,Idaho
South Atlantic	Roanoke,Virginia	Roanoke,Virginia	Mingo,West Virginia

large. The adjustment factor does not affect IMI_u only in situations where the landscape is a perfect circle and there are no urban areas. When there are no urban areas, IMI_u is 0 anyway.

Data preparation

I use National Land Cover Data (NLCD), circa 2011, was produced by the U.S. Geological Survey (2014) and retrieved from the Multi-Resolution Land Characteristics Consortium website ($\sim$ 9 billion pixels, 30 m resolution). The data is selected because of it suitability to demonstrate the indices on wide range of urban patterns rather than its currency. I focus only on the urban land cover classes (21–24) at a county level, for the contiguous United States ($n = 3,109$). This urban land cover is noisy because of presence of roads and other linear features that impact fragmentation and compactness metrics. I use procedures described in Kaza (2020) (e.g. morphological operations, removal of small patches etc.) to prepare an adjusted urban land cover for each county.

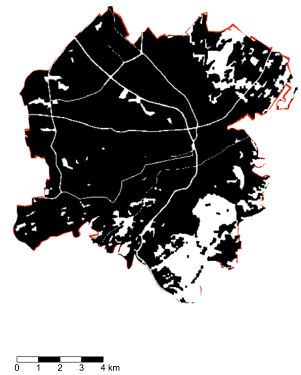
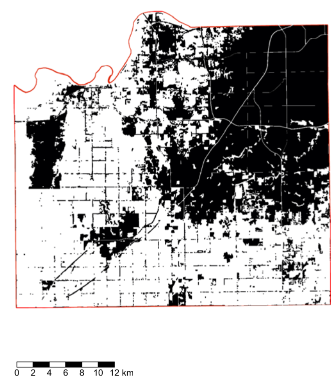
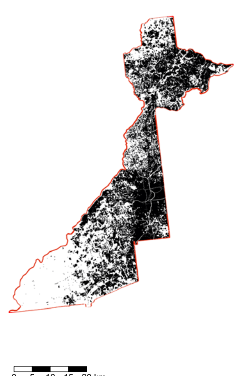
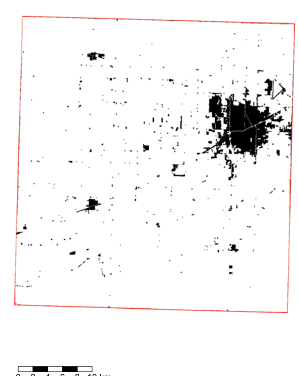
To determine the areas of high slopes, I use elevation data from the U.S. Geological Survey (Sugarbaker et al. 2017) and determine pixels of more than 15% slope. Water features are derived from the NLCD water and perennial snow classes (11,12). County boundaries are from U.S. Census, downloaded using tigris package (Walker 2019) and rasterized using fasterize package (Ross 2018).

Discussion

According to IMI_u, Dallas and Orange are the most compact counties in West South Central and Pacific Census Divisions (see Table 1). The reputation that Dallas-Fort Worth and Los Angeles Metropolitan areas (of which these counties are part of) as the quintessential sprawling metropolitan areas poses questions about external validity for the use of IMI_u as a measure of urban compactness. Once landscape adjustment is made, IMI_{adj} returns Multnomah County (Portland) and Orleans Parish (New Orleans) as the most compact county in the Pacific and West South Central Census Divisions respectively. Jefferson County, Washington sees the largest gain in IMI due to the adjustment (while IMI_u is 0.01, IMI_{adj} is 0.60) in the Pacific Division. The county is home to the Olympic mountains and the central part the county is uninhabited. The eastern and western communities are not connected by road due to the barriers. Such urban form, predictably is penalized by IMI_u, the adjustment recognizes these limitations and alleviates the score. The proposed adjustment, also substantially changes the scores of three counties in Fig. 1; New York ($IMI_u = 0.27, IMI_{adj} = 0.65$), Multnomah ($IMI_u = 0.53, IMI_{adj} = 0.84$) and Rock Island ($IMI_u = 0.20, IMI_{adj} = 0.60$).

Similarly, Suffolk (Long Island), Fulton (Atlanta) have non-compact county boundaries and therefore receive large adjustments (see Fig. 2). Jefferson Parish in Louisiana have significant water features that separate the main urban areas, also is now pegged as moderately compact. On the other hand, already compact counties (e.g.Roanoke) or relatively developmentally unrestricted counties (e.g. Johnson and

Fig. 2 Illustration of changes in the score because of adjustment for different urban patterns

Large Changes	Small Changes
	
Suffolk, New York $IMI_u = 0.221$ $IMI_{adj} = 0.623$ $IMI_\Delta = 0.402$	Ronoake, Virginia $IMI_u = 0.838$ $IMI_{adj} = 0.851$ $IMI_\Delta = 0.014$
	
Jefferson, Louisiana $IMI_u = 0.171$ $IMI_{adj} = 0.755$ $IMI_\Delta = 0.584$	Johnson, Kansas $IMI_u = 0.551$ $IMI_{adj} = 0.574$ $IMI_\Delta = 0.023$
	
Fulton, Georgia $IMI_u = 0.202$ $IMI_{adj} = 0.610$ $IMI_\Delta = 0.407$	Hall, Nebraska $IMI_u = 0.128$ $IMI_{adj} = 0.143$ $IMI_\Delta = 0.016$

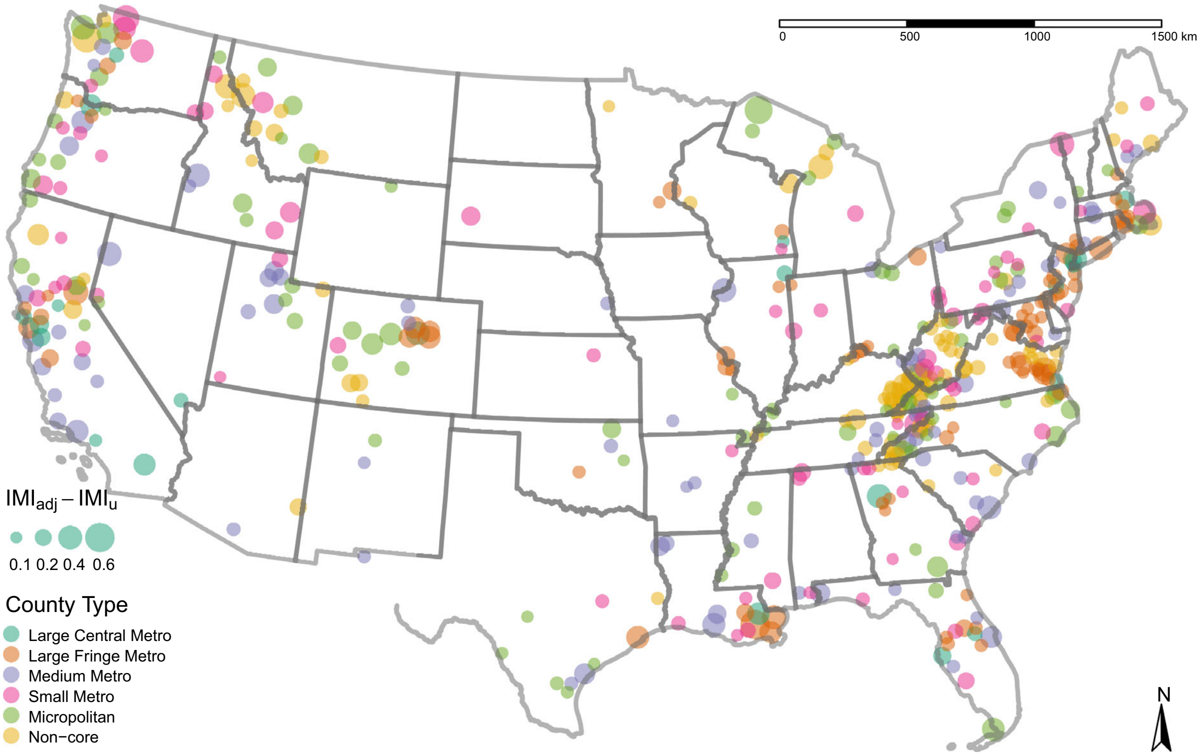


Fig. 3 Change in the compactness index of urban areas because of landscape adjustment. Only counties with more than 0.1 differential are shown for illustration purposes. The full dataset is available at https://doi.org/10.15139/S3/YLZEH4

Hall) experience very little changes in the score (see Fig. 2).

A vast majority of the counties experience very little change in the urban compactness score. Only 14% of the counties see their IMI go up by more than 0.1 and 2% see changes that are more than 0.2. But in some instances, the score is adjusted by as much as 200%. Most of the counties that experience large changes are located in the heavily urbanized Northeastern corridor (see Fig. 3), in the hilly regions of Appalachia, in the Bayous in the Gulf coast of Louisiana. In the Pacific coast where compactness score is improved, it is due to mountainous and irregularly shaped counties. Another distinct cluster can be observed in counties along the Rocky mountains in Idaho and Colorado. Coastal counties that are constrained in their development patterns by water in South Carolina and Florida also experience modest changes.

Counties are categorized by their metropolitan status (metro vs non-metro), size (large, medium and small) and location (central and fringe) by National Center for Health Statistics (NCHS) (Ingram and Franco 2012). Only 22 out of the 68 large central metros experience changes in IMI over 0.1. However, these changes are significant. IMI_{adj} for New York county is 0.65, while IMI_u is 0.27. Similarly, Fulton, Georgia (Atlanta), Denver and Riverside counties experience modest to large gains. While this adjustment seems to provide defensible indicators for New York, the case of Fulton reminds us that IMI does not capture the effect of population density. It is a measure of dispersion and fragmentary urban patterns in two dimensions.

22% of other types of metro counties experienced significant change because of the adjustment. For example, the large fringe metro counties such as Jefferson Parish, Louisiana saw large gains, as did Placer County, California and Suffolk County, New York (Long Island). Some, non-core counties also saw significant gains rivalling those of large fringe metro counties, but their gains are not substantively important as the total amount of urbanization in non-core counties is small and is not as important.

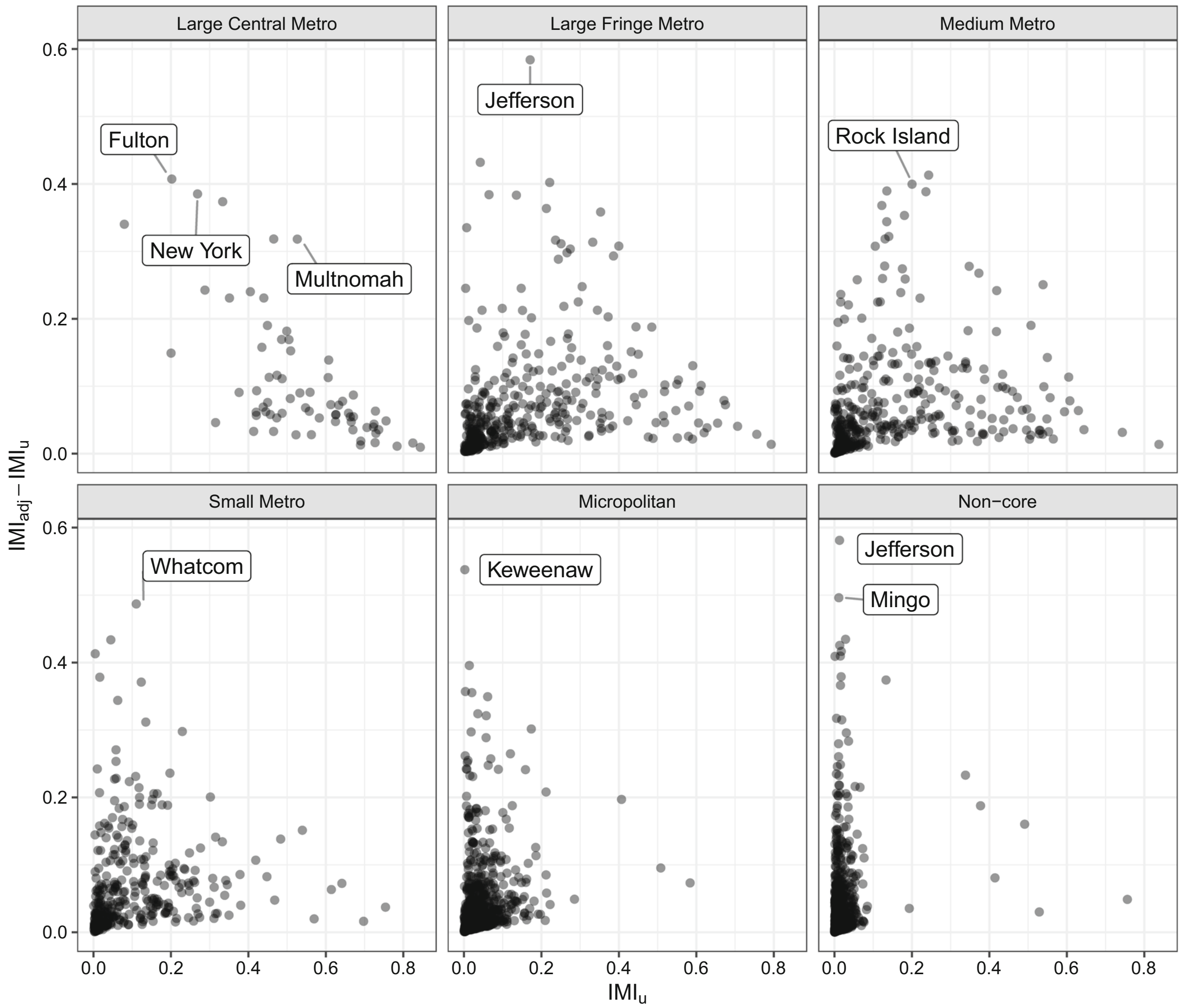


Fig. 4 Change in urban IMI due to landscape adjustments relative to unadjusted compactness scores for different types of counties

The scatter plot of the IMI_{Δ} relative to IMI_u reveal that the changes are most significant at the lower end of the urban compactness score (see Fig. 4), when they are significant. The more compact counties, the smaller the adjustment as can be expected. However, in large central counties, some of moderately compact counties experience substantial changes in the score. In medium and small metro counties the adjustment has moderate effect on more non-compact counties, though there are some medium metro counties in the middle of the spectrum that experience changes over 0.2. In micropolitan and non-core counties, almost exclusively the large effects are found in the lower end of IMI_u distribution.

Comparison with other metrics

In this subsection, I demonstrate the performance of IMI_{adj} relative to Angel et al. (2010)'s EI and Taubenböck et al. (2019)'s DI. To make comparisons more robust and straightforward, I linearly scale DI to [0,1] and use $1 - \mathrm{DI}$ as a measure of compactness. In all the three cases, compact urban form gets values closer to 1, while dispersed urban form is closer to 0.

The distribution of $1 - \mathrm{DI}$ does not capture the range of urban patterns. The inter quartile range for the index is less than 0.01. Only 34 of the 3109 counties have the index more than 0.6. Most of these counties are compact cities with small geography and tightly connected contiguous urban areas such as Baltimore

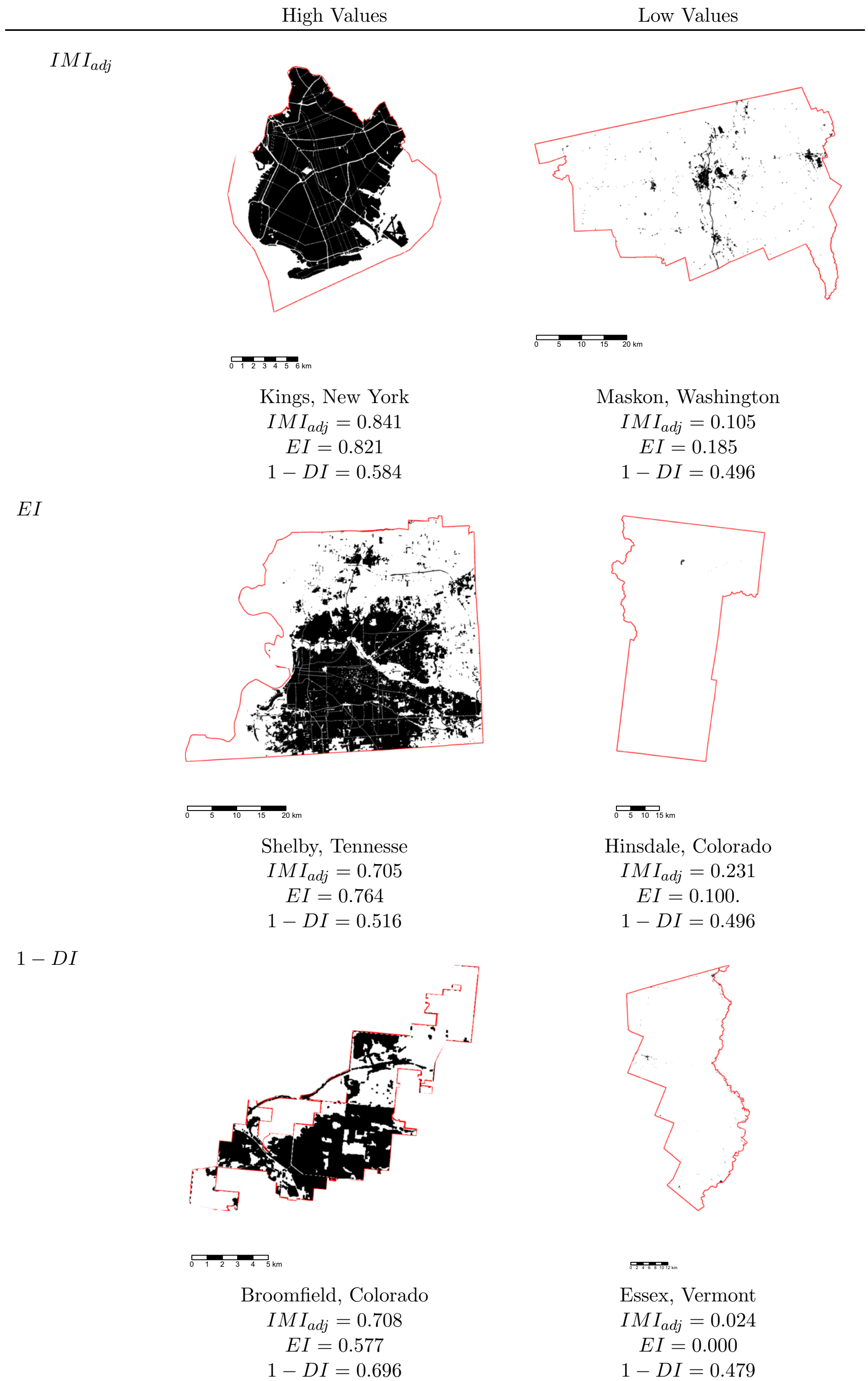


Fig. 5 Illustration of the performance of different compactness indicators. $\mathrm{IMI}_{\mathrm{adj}}$ is the Adjusted Index of Moment of Inertia. *EI* is the Exchange Index. *DI* is the Dispersion Index

city and Broomfield county (see Fig. 5). This is because the largest patch index penalises discontinuous but tightly clustered urban form. On the other hand, outliers in the index for the non-core counties are predominantly dominated by small independent cities (county equivalents) in Virginia. While 1 – DI

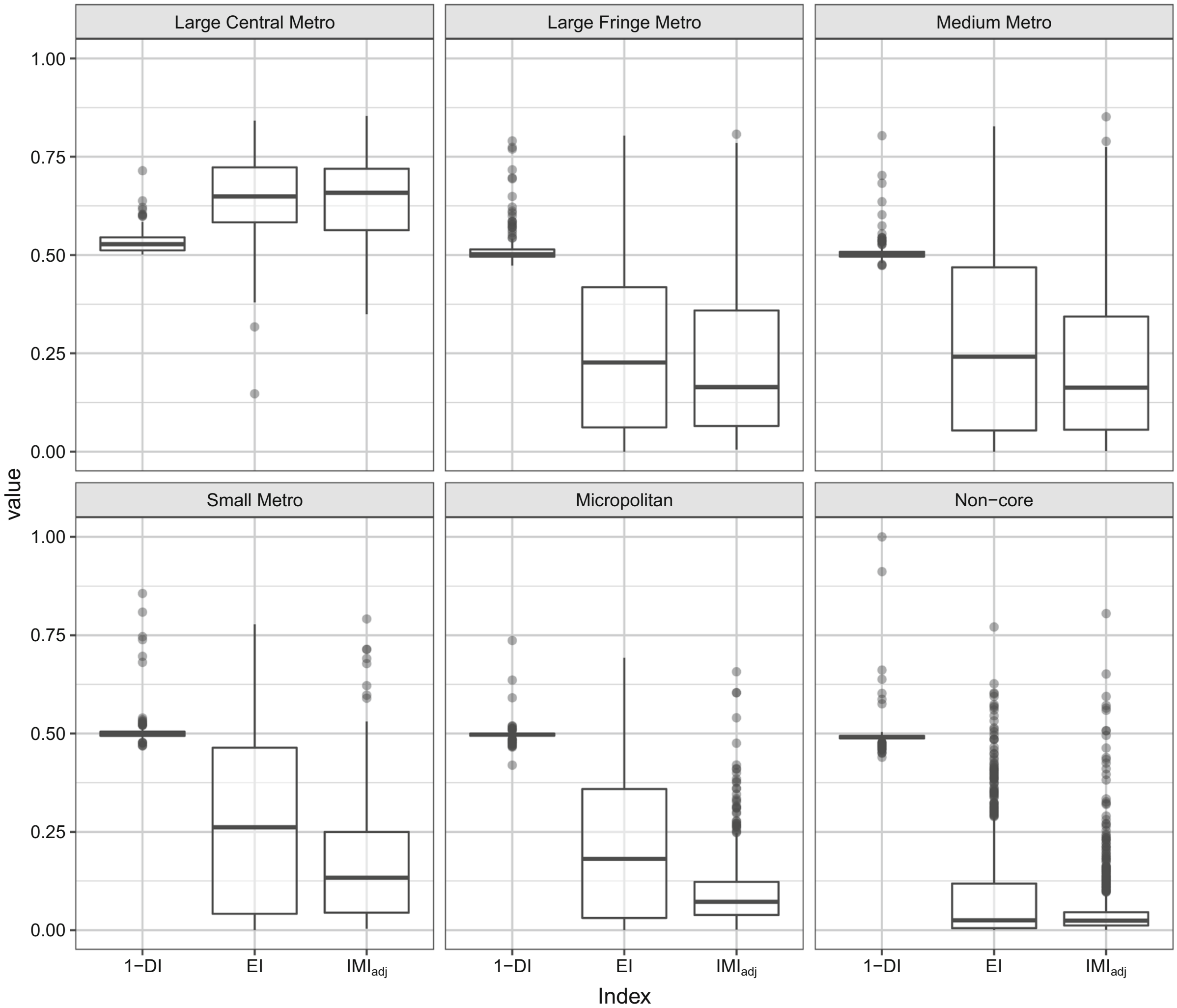


Fig. 6 Comparision of different compactness indicators by county type. IMI_{adj} is the Adjusted Index of Moment of Inertia. EI is the Exchange Index. DI is the Dispersion Index

might be useful to measure compactness of urban areas at a metropolitan scale for mega cities, the measure is not informative to measure urban form at county scale in the United States.

The distribution of the EI is much more similar to that of IMI. The Spearman correlation between the two indices is 0.77, which suggests modest correlation. However, it should be noted that 18% of the counties have no EI index because the EAC does not intersect with the urban areas. This is because the centroid of the urban areas fall in areas that are not urbanised. Vast majority (75%) of these counties are non-core but 77 of these counties are metropolitan counties. However, it should be noted that predominantly rural counties such as Gates, Virginia and Gilpin, Colorado are included in these metropolitan characterisations, suggesting caution in uncritically accepting the NCHS county type classification.

While all three indices characterise large central metro counties as more compact (see Fig. 6) than other types of counties. The spread of the indices is also much lower for these large central metro counties. However, EI characterises other types of metro counties more compact than IMI, on average. This is because IMI penalises distance of urban areas from the centroid, where as EI only accounts for whether the urban areas are within the EAC or not.

Conclusions

In this short note, I demonstrated that identifying compactness of urban area is complicated by landscape shape and other restrictions. I demonstrated an adjustment to the IMI that account for both coverage ratio and elongated landscape shapes. While the adjustment for landscape shape makes the IMI more representative of the compactness of urban form, it does not fully capture our intuitive understanding of compact urban form. Thus, the index should be used in conjunction with other indicators to fully capture the richness of the multi-dimensionality of urban condition.

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